Special Factors

$(x + y)^2 = x^2 + 2xy + y^2$ | $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ | $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$x^2 - y^2 = (x + y)(x - y)$ | $(x - y)^2 = x^2 - 2xy + y^2$ | $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ | $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

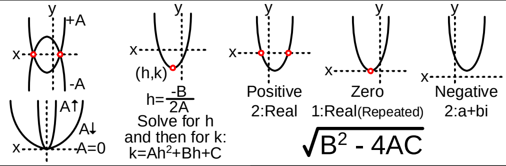
Radians = Degrees $\cdot \frac{\pi}{180}$

Degrees = Radians $\cdot \frac{180}{\pi}$

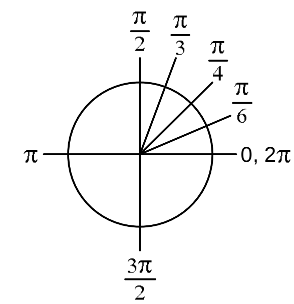
Quadratic Formula

Where: A>0, $Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$



$\sqrt{B^2 - 4AC}$



Deg	0	30	45	60	90
Rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
Sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

Sin = y | Sin(-x) = -Sin(x)

Cos = x | Cos(-x) = -Cos(x)

Tan = y/x | Tan(-x) = -Tan(x)

Csc = 1/Sin | Csc(-x) = -Csc(x)

Sec = 1/Cos | Sec(-x) = -Sec(x)

Cot = 1/Tan | Cot(-x) = -Cot(x)

$Csc^2 = 1/Sin^2$

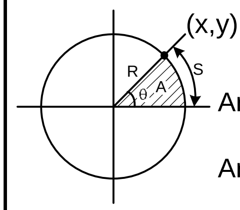
$Sec^2 = 1/Cos^2$

$Cot^2 = 1/Tan^2$

$Sin^2 + Cos^2 = 1$ (A)

$Sec^2 - Tan^2 = 1$

$Csc^2 - Cot^2 = 1$



$S = R\theta$

Area $= \frac{1}{2}R^2\theta$

Area $= \frac{1}{2}RS$

$Sin(x+y) = Sin(x)Cos(y) + Sin(y)Cos(x)$

$Sin(x-y) = Sin(x)Cos(y) - Sin(y)Cos(x)$

$Cos(x+y) = Cos(x)Cos(y) - Sin(x)Sin(y)$

$Cos(x-y) = Cos(x)Cos(y) + Sin(x)Sin(y)$

$$Tan(x+y) = \frac{Tan(x) + Tan(y)}{1 - Tan(x)Tan(y)}$$

$$Tan(x-y) = \frac{Tan(x) - Tan(y)}{1 + Tan(x)Tan(y)}$$

Graphing

Y = ASin(Bx+C)+D **Y = ACos(Bx+C)+D**		**Y = ATan(Bx+C)+D** **Y = ACot(Bx+C)+D**	
Endpoints of 1 cycle		Endpoints of 1 cycle	
Left:	0 = (Bx+C)	Left:	$\frac{-\pi}{2}$ = (Bx+C)
Right:	2π = (Bx+C)	Right:	$\frac{\pi}{2}$ = (Bx+C)
Period:	$\frac{2\pi}{\lvert B\rvert}$	Period:	$\frac{\pi}{\lvert B\rvert}$
Phase shift:	$\frac{-C}{B}$	Phase shift:	$\frac{-C}{B}$
Amplitude (pk):	|A|	Rate of rise:	|A|
Y Offset	D	Y Offset:	D

$Sin(2x) = 2Sin(x)Cos(x)$

$Cos(2x) = Cos^2(x) - Sin^2(x)$ (B)

$Cos(2x) = 2Cos^2(x) - 1$

$Cos(2x) = 1 - 2Sin^2(x)$

$2Sin^2(x) = 1 - Cos(2x)$

$2Cos^2(x) = 1 + Cos(2x)$

These are derived by substituting (A) into (B)

$$Tan(2x) = \frac{2Tan(x)}{1 - Tan^2(x)}$$

$$Sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-Cos(x)}{2}}$$

$$Cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+Cos(x)}{2}}$$

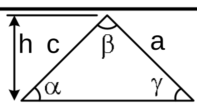
$$Tan\left(\frac{x}{2}\right) = \frac{Sin(x)}{1+Cos(x)} = \frac{1-Cos(x)}{Sin(x)}$$

$$Sin(x)Sin(y) = \frac{1}{2}\left[Cos(x-y) - Cos(x+y)\right]$$

$$Cos(x)Cos(y) = \frac{1}{2}\left[Cos(x-y) + Cos(x+y)\right]$$

$$Sin(x)Cos(y) = \frac{1}{2}\left[Sin(x+y) - Sin(x-y)\right]$$

Law of Sines/Cosines



$$\frac{Sin(\alpha)}{a} = \frac{Sin(\beta)}{b} = \frac{Sin(\gamma)}{c}$$

$a^2 = b^2 + c^2 - 2bcCos(\alpha)$

$b^2 = a^2 + c^2 - 2acCos(\beta)$

$c^2 = a^2 + b^2 - 2abCos(\gamma)$

b x h Formula Area $= \frac{1}{2}b \cdot h$

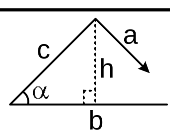
Sine Formula Area $= \frac{c \cdot b \cdot Sin(\alpha)}{2}$

Herons Formula $S = \frac{a+b+c}{2}$

Area $= \sqrt{S(S-a)(S-b)(S-c)}$

Given: a, c and α

Calc: $h = c \cdot Sin(\alpha)$



a<h:	0 solutions
a=h:	1 solution
a≥c:	1 solution
h<a<c:	2 solutions

c: Fixed side

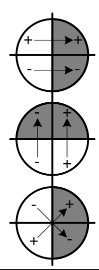
a: Swinging side

Inverse functions defined as...

$y = Sin^{-1}(x)$ for $x\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$y = Cos^{-1}(x)$ for $x\left(0, \pi\right)$

$y = Tan^{-1}(x)$ for $x\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



Complex number powers/roots

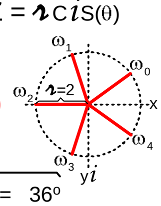
Z raised to the power of n $\quad Z = r\,cis(\theta)$

$Z^n = r^n cis(n\theta)$

The n x n^{th} roots of Z

$$\sqrt[n]{Z} = \sqrt[n]{r}\,cis\left(\frac{\theta+2\pi k}{n}\right) \text{ or } \left(\frac{\theta+360^\circ k}{n}\right)$$

$$\sqrt[5]{Z} = \omega_k = \sqrt[5]{32}\,cis\left(\frac{\pi+2\pi k}{5}\right)$$



$\omega_0 = 2\,cis\left(\frac{\pi}{5}\right)$ $\quad k_0: \frac{\pi}{5} = \frac{\pi}{5} = 36^\circ$

$\omega_1 = 2\,cis\left(\frac{3\pi}{5}\right)$ $\quad k_1: \frac{\pi+2\pi}{5} = \frac{3\pi}{5} = 108^\circ$

$\omega_2 = 2\,cis\left(\frac{5\pi}{5}\right)$ $\quad k_2: \frac{\pi+4\pi}{5} = \frac{5\pi}{5} = 180^\circ$

$\omega_3 = 2\,cis\left(\frac{7\pi}{5}\right)$ $\quad k_3: \frac{\pi+6\pi}{5} = \frac{7\pi}{5} = 252^\circ$

$\omega_4 = 2\,cis\left(\frac{9\pi}{5}\right)$ $\quad k_4: \frac{\pi+8\pi}{5} = \frac{9\pi}{5} = 324^\circ$

Complex number operations

$i = \sqrt{-1}$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

etc

Rectangular=Geometric

$Z = a + bi$

Polar=Trigonometric

$Z = r(Cos(\theta) + iSin(\theta))$

or

$Z = r\,cis(\theta)$

Rect → Polar

$r = |Z| = \sqrt{a^2+b^2}$

$\theta = Tan^{-1}\left(\frac{b}{a}\right)$

Polar → Rect

$a = r\,Cos(\theta)$

$b = r\,Sin(\theta)$

θ is always (0-2π)!

$Z_1 = a + bi \quad Z_2 = c + di$

$Z_1 + Z_2 = (a+c) + (b+d)i$

$Z_1 - Z_2 = (a-c) + (b-d)i$

$Z_1Z_2 = (a+bi)(c+di)$

$$\frac{Z_1}{Z_2} = \frac{(a+bi)}{(c+di)}$$

$$= \frac{(a+bi)}{(c+di)} \cdot \frac{(c-di)}{(c-di)}$$

Multiply top/bot by conjugate of denominator

$$= \frac{(a+bi)(c-di)}{c^2+d^2}$$

FOIL top, bottom is sum of squares.

Finish: $m = c^2 + d^2$ $\quad \frac{x}{m} + \frac{yi}{m}$

$Z_1 = r_1 cis(\theta_1) \quad Z_2 = r_2 cis(\theta_2)$

$Z_1Z_2 = r_1r_2 cis(\theta_1 + \theta_2)$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2)$$

For $Tan^{-1}\left(\frac{b}{a}\right)$